whether it is possible to say how large a part of the space occupied by the whole molecule is occupied by the atoms; but perhaps the atoms bear to the molecule some such relationship as the molecule to the drop of water referred to. Finally, the corpuscles may stand to the atom in a similar scale of magnitude. Accordingly, a threefold magnification would be needed to bring these ultimate parts of the atom within the range of our ordinary scales of measurement.

I have already considered what would be observed under the triply powerful microscope, and must now return to the intermediate stage of magnification, in which we consider those communities of atoms which form molecules. This is the field of research of the chemist. Although prudence would tell me that it would be wiser not to speak of a subject of which I know so little, yet I can not refrain from saying a few words.

The community of atoms in water has been compared with a triple star, but there are others known to the chemist in which the atoms are to be counted by fifties and hundreds, so that they resemble constellations.

I conceive that here again we meet with conditions similar to those which we have supposed to exist in the atom. Communities of atoms are called chemical combinations, and we know that they possess every degree of stability. The existence of some is so precarious that the chemist in his laboratory can barely retain them for a moment; others are so stubborn that he can barely break them up. In this case dissociation and reunion into new forms of communities are in incessant and spontaneous progress throughout the world. The more persistent or more stable combinations succeed in their struggle for life, and are found in vast quantities, as in the cases of common salt and of the combinations of silicon. But no one has ever found a mine of guncotton, because it has so slight a power of resistance. If, through some accidental collocation of elements, a single molecule of guncotton were formed, it would have but a short life.

Stability is, further, a property of relationship to surrounding conditions; it denotes adaptation to environment. Thus salt is adapted to the struggle for existence on the earth, but it can not withstand the severer conditions which exist in the sun.

G. H. Darwin.
University of Cambridge.

[The president here announced that he proposed to consider various theories of evolution in the heavens in the second portion of his address, to be delivered at Johannesburg on Wednesday, August 30.]

ADDRESS TO THE MATHEMATICAL AND PHYSICAL SECTION OF THE BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE.[1]

According to an established and unchallenged custom, our proceedings are inaugurated by an address from the president. Let me begin it by discharging a duty which, unhappily, is of regular recurrence. If your president only mentions names when he records the personal losses suffered during the year by the sciences of the section, the corporate sense of the section will be able to appreciate the losses with a deeper reality than can be conveyed by mere words.

In Mr. Ronald Hudson, who was one of our secretaries at the Cambridge meeting a year ago, we have lost a mathematician whose youthful promise had ripened into early performance. The original work which he had accomplished is sufficient, both in quality and in amount, to show that much has been given, and that much more

[1] South Africa, 1905.

could have been expected. His alert and bright personality suggested that many happy years lay before him. All these fair hopes were shattered in a moment by an accident upon a Welsh hillside; and his friends, who were many, deplore his too early death at the age of twenty-eight.

The death of Mr. Frank McClean has robbed astronomy of one of its most patient workers and actively creative investigators. I wish that my own knowledge could enable me to give some not inadequate exposition of his services to the science which he loved so well. He was a man of great generosity which was wise, discriminating and more than modest; to wide interests in science he united wide interests in the fine arts. Your astronomer royal, in the Royal Observatory at Cape Town, will not lightly forget his gift of a great telescope: and the University of Cambridge, the grateful recipient of his munificent endowment of the Isaac Newton studentships fifteen years ago, and of his no less munificent bequest of manuscripts, early printed books, and objects of art, has done what she can towards perpetuating his memory for future generations by including his name in the list, that is annually recited in solemn service, of her benefactors who have departed this life.

In the early days of our gatherings, when the set of cognate sciences with which we specially are concerned had not yet diverged so widely from one another alike in subject and in method, this inaugurating address was characterized by a brevity that a president can envy and by a freedom from formality that even the least tolerant audience could find admirable. The lapse of time, perhaps assisted by presidential ambitions which have been veiled under an almost periodic apology for personal shortcomings, has deprived these addresses of their ancient brevity, and has invested them with an air of oracular gravity. The topics vary from year to year, but this variation is due to the predilection of the individual presidents; the types of address are but few in number. Sometimes, indeed, we have had addresses that can not be ranged under any comprehensive type. Thus one year we had an account of a particular school of long-sustained consecutive research; another year the president made a constructive (and perhaps defiant) defence of the merits of a group of subjects that were of special interest to himself. But there is one type of address which recurs with iterated frequency; it is constituted by a general account of recent progress in discovery, or by a survey of modern advances in some one or other of the branches of science to which the multiple activities of our section are devoted. No modern president has attempted a general survey of recent progress in all the branches of our group of sciences; such an attempt will probably be deferred until the council discovers a president who, endowed with the omniscience of a Whewell, and graced with the tongue of men and of angels, shall once again unify our discussions.

On the basis of this practise, it would have been not unreasonable on my part to have selected some topic from the vast range of pure mathematics, and to have expounded some body of recent investigations. There certainly is no lack of topics; our own day is peculiarly active in many directions. Thus, even if we leave on one side the general progress that has been made in many of the large branches of mathematics during recent years, it is easy to hint at numerous subjects which could occupy the address of a mathematical president. He might, for instance, devote his attention to modern views of continuity, whether of quantity or of space; he might be heterodox or orthodox as to the so-called laws of motion; he might expound his

notions as to the nature and properties of analytic functionality; a discussion of the hypotheses upon which a consistent system of geometry can be framed could be made as monumental as his ambition might choose; he could revel in an account of the most recent philosophical analysis of the foundations of mathematics, even of logic itself, in which all axioms must either be proved or be compounded of notions that defy resolution by the human intellect at the present day. Such discussions are bound to be excessively technical unless they are expressed in unmathematical phraseology; when they are so expressed, and in so far as such expression is possible, they become very long and they can be very thin. Moreover, had I chosen any topic of this character, it would have been the merest natural justice to have given early utterance of the sibyllic warning to the uninitiated; I must also have bidden the initiated that, as they come, they should summon all the courage of their souls. So I abstain from making such an experiment upon an unwarned audience; yet it is with reluctance that I have avoided subjects in the range which to me is of peculiar interest.

On the other hand, I must ask your indulgence for not conforming to average practice and expectation. My desire is to mark the present occasion by an address of unspecialized type which, while it is bound to be mainly mathematical in tenor, and while it will contain no new information, may do little more than recall some facts that are known, and will comment briefly upon obvious tendencies. Let me beg you to believe that it is no straining after novelty which has dictated my choice; such an ambition has a hateful facility of being fatal both to the performer and to the purpose. It is the strangeness of our circumstances, in both place and time, that has suggested my subject. With an adventurous audacity that quite overcrows the spirit of any of its past enterprises, the British Association for the Advancement of Science has traveled south of the Equator and, in accepting your hospitality, proposes to traverse much of South Africa. The prophet of old declared that 'many shall run to and fro, and knowledge shall be increased'; if the second part of the prophecy is not fulfilled, it will not be for the want of our efforts to fulfill the first part. And if the place and the range of this peripatetic demonstration of our annual corporate activity are unusual, the occasion chosen for this enterprise recalls memories that are fundamental in relation to our subject. It is a modern fashion to observe centenaries. In this section we are in the unusual position of being able to observe three scientific centenaries in one and the same year. Accordingly I propose to refer to these in turn, and to indicate a few of the events filling the intervals between them; but my outline can be of only the most summary character, for the scientific history is a history of three hundred years, and, if searching enough, it could include the tale of nearly all mathematical and astronomical and physical science.

It is exactly three hundred years since Bacon published 'The Advancement of Learning.' His discourse, alike in matter, in thought, in outlook, was in advance of its time, and it exercised no great influence for the years that immediately followed its appearance; yet that appearance is one of the chief events in the origins of modern natural science. Taking all knowledge to be his province, he surveys the whole of learning: he deals with the discredits that then could attach to it; he expounds both the dignity and the influence of its pursuit; and he analyzes all learning, whether of things divine or of things human, into

its ordered branches. He points out deficiencies and gaps; not a few of his recommendations of studies, at his day remaining untouched, have since become great branches of human thought and human inquiry. But what concerns us most here is his attitude towards natural philosophy, all the more remarkable because of the state of knowledge of that subject in his day, particularly in England. It is true that Gilbert had published his discovery of terrestrial magnetism some five years earlier, a discovery followed only too soon by his death; but that was the single considerable English achievement in modern science down to Bacon's day.

In order to estimate the significance of Bacon's range of thought let me recite a few facts, as an indication of the extreme tenuity of progressive science in that year (1605). They belong to subsequent years, and may serve to show how restricted were the attainments of the period, and how limited were the means of advance. The telescope and the microscope had not yet been invented. The simple laws of planetary motion were not formulated, for Kepler had them only in the making. Logarithms were yet to be discovered by Napier, and to be calculated by Briggs. Descartes was a boy of nine and Fermat a boy of only four, so that analytical geometry, the middle-life discovery of both of them, was not yet even a dream for either of them. The Italian mathematicians, of whom Cavalieri is the least forgotten, were developing Greek methods of quadrature by a transformed principle of indivisibles; but the infinitesimal calculus was not really in sight, for Newton and Leibnitz were yet unborn. Years were to elapse before, by the ecclesiastical tyranny over thought, Galileo was forced to make a verbal disavowal of his adhesion to the Copernican system of astronomy, of which he was still to be the protagonist in propounding any reasoned proof. Some mathematics could be had, cumbrous arithmetic and algebra, some geometry lumbering after Euclid, and a little trigonometry; but these were mainly the mathematics of the Renaissance, no very great advance upon the translated work of the Greeks and the transmitted work of the Arabs. Even our old friend the binomial theorem, which now is supposed to be the possession of nearly every able schoolboy, remained unknown to professional mathematicians for more than half a century yet to come.

Nor is it merely on the negative side that the times seemed unpropitious for a new departure; the spirit of the age in the positive activities of thought and deed was not more sympathetic. Those were the days when the applications of astronomy had become astrology. Men sought for the elixir of life and pondered over the transmutation of baser metals into gold. Shakespeare not long before had produced his play 'As You Like It,' where the strange natural history of the toad which,

> Ugly and venomous,
> Bears yet a precious jewel in his head,

is made a metaphor to illustrate the sweetening uses of adversity. The stiffened Elizabethan laws against witchcraft were to be sternly administered for many a year to come. It was an age that was pulsating with life and illuminated by fancy, but the life was the life of strong action and the fancy was the fancy of ideal imagination; men did not lend themselves to sustained and abstract thought concerning the nature of the universe. When we contemplate the spirit that such a state of knowledge might foster towards scientific learning, and when we recall the world into which Bacon's treatise was launched, we can well be surprised at his far-reaching views, and we can marvel at his isolated wisdom.

Let me select a few specimens of his judgments, chosen solely in relation to our own subjects. When he says

All true and fruitful natural philosophy hath a double scale or ladder, ascendent and descendent, ascending from experiments to the invention of causes, and descending from causes to the invention of new experiments; therefore I judge it most requisite that these two parts be severally considered and handled—

he is merely expounding, in what now is rather archaic phrase, the principles of the most ambitious investigations in the natural philosophy of subsequent centuries. When he speaks of

the operation of the relative and adventive characters of essences, as quantity, similitude, diversity, possibility and the rest; with this distinction and provision, that they be handled as they have efficacy in nature, and not logically—

I seem to hear the voice of the applied mathematician warning the pure mathematician off the field. When, after having divided natural philosophy into physic and metaphysic (using these words in particular meanings, and including mathematics in the second of the divisions), he declares

physics should contemplate that which is inherent in matter, and therefore transitory, and metaphysics that which is abstracted and fixed; * * * physic describeth the causes of things, but the variable or respective causes; and metaphysic the fixed and constant causes—

there comes before my mind the army of physicists of the present day, who devote themselves unwearyingly to the properties of matter and willingly cast aside elaborate arguments and calculations. When he argues that

many parts of nature can neither be invented with sufficient subtilty, nor demonstrated with sufficient perspicuity, nor accommodated unto use with sufficient dexterity, without the aid and intervening of the mathematics—

he might be describing the activity of subsequent generations of philosophers, astronomers and engineers. And in the last place (for my extracts must have some end), when he expresses the opinion

that men do not sufficiently understand the excellent use of the pure mathematics, in that they do remedy and cure many defects in the wit and faculties intellectual. For if the wit be too dull, they sharpen it; if too wandering, they fix it; if too inherent in the sense, they abstract it; * * * in the mathematics, that which is collateral and intervenient is no less worthy than that which is principal and intended—

I seem to hear an advocate for the inclusion of elementary mathematics in any scheme of general education. At the same time, I wonder what Bacon, who held such an exalted estimate of pure mathematics in its gray dawn, would have said by way of ampler praise of the subject in its fuller day.

It was a splendid vision of inductive science as of other parts of learning: it contained a revelation of the course of progress through the centuries to come. Yet the facts of to-day are vaster than the vision of that long-ago yesterday, and human activity has far outstripped the dreams of Bacon's opulent imagination. He was the harbinger (premature in many respects it must be confessed, but still the harbinger) of a new era. At a time when we are making a new departure in the fulfilment of the purpose of our charter, which requires us 'to promote the intercourse of those who cultivate science in different parts of the British Empire, our Association for the Advancement of Science may pause for a moment to gaze upon the vision revealed three centuries ago in the 'Advancement of Learning' by a philosopher whose influence upon the thought of the world is one of the glories of our nation.

I have implied that Bacon's discourse was in advance of its age, so far as England was concerned. Individuals could make their mark in isolated fashion. Thus Harvey, in his hospital work in London, discovered the circulation of the blood; Na-

pier, away on his Scottish estates, invented logarithms; and Horrocks, in the seclusion of a Lancashire curacy, was the first to observe a transit of Venus. But for more than half a century the growth of physical science was mainly due to workers on the continent of Europe. Galileo was making discoveries in the mechanics of solids and fluids, and, specially, he was building on a firm foundation the fabric of the system of astronomy, hazarded nearly a century before by Copernicus; he still was to furnish, by bitter experience, one of the most striking examples in the history of the world that truth is stronger than dogma. Kepler was gradually elucidating the laws of planetary motion, of which such significant use was made later by Newton; and Descartes, by his creation of analytical geometry, was yet to effect such a constructive revolution in mathematics that he might not unfairly be called the founder of modern mathematics. In England the times were out of scientific joint: the political distractions of the Stuart troubles, and the narrow theological bitterness of the commonwealth, made a poor atmosphere for the progress of scientific learning, which was confined almost to a faithful few. The fidelity of those few, however, had its reward; it was owing to their steady confidence and to their initiative that the Royal Society of London was founded in 1662 by Charles II. At that epoch, science (to quote the words of a picturesque historian) became the fashion of the day. Great Britain began to contribute at least her fitting share to the growing knowledge of nature; and her scientific activity in the closing part of the seventeenth century was a realization, wonderful and practical, of a part of Bacon's dream. Undoubtedly the most striking contribution made in that period is Newton's theory of gravitation, as expounded in his 'Principia,' published in 1687.

That century also saw the discovery of the fluxional calculus by Newton, and of the differential calculus by Leibnitz. These discoveries provided the material for one of the longest and most deadening controversies as to priority in all the long history of those tediously barren occupations; unfortunately they are dear to minds which cannot understand that a discovery should be used, developed, amplified, but should not be a cause of envy, quarrel, or controversy. Let me say, incidentally, that the controversy had a malign influence upon the study of mathematics as pursued in England.

Also, the undulatory theory of light found its first systematic, if incomplete, exposition in the work of Huygens before the century was out. But Newton had an emission theory of his own, and so the undulatory theory of Huygens found no favor in England until rather more than a hundred years later; the researches of Thomas Young established it on a firm foundation.

Having thus noted some part of the stir in scientific life which marked the late years of the seventeenth century, let me pass to the second of our centenaries: it belongs to the name of Edmond Halley. Quite independently of his achievement connected with the year 1705 to which I am about to refer, there are special reasons for honoring Halley's name in this section at our meeting in South Africa. When a young man of twenty-one he left England for St. Helena, and there, in the years 1676–1678, he laid the foundations of stellar astronomy for the southern hemisphere; moreover, in the course of his work he there succeeded in securing the first complete observation of a transit of Mercury. After his return to England, the next few years of his life were spent in laying science under a special debt that can hardly be over-appreciated. He placed himself in personal relation with Newton, propounded to him questions and

offered information; and it is now a commonplace statement that Halley's questions and suggestions caused Newton to write the 'Principia.' More than this, we know that Newton's great treatise saw the light only through Halley's persuasive insistence, through his unwearying diligence in saving Newton all cares and trouble and even pecuniary expense, and through his absolutely self-sacrificing devotion to what he made an unwavering duty at that epoch in his life. Again, he appears to have been the first organizer of a scientific expedition, as distinct from a journey of discovery, towards the southern seas: he sailed as far as the fifty-second degree of southern latitude, devised the principle of the sextant in the course of his voyaging, and, as a result of the voyage, he produced a general chart of the Atlantic Ocean, with special reference to the deviation of the compass. Original, touched with genius, cheery of soul, strenuous in thought and generous by nature, he spent his life in a continuously productive devotion to astronomical science, from boyhood to a span of years far beyond that which satisfied the psalmist's broodings. I have selected a characteristic incident in his scientific activity, one of the most brilliant (though it can not be claimed as the most important) of his astronomical achievements; it strikes me as one of the most chivalrously bold acts of convinced science within my knowledge. It is only the story of a comet.

I have just explained, very briefly, Halley's share in the production of Newton's 'Principia'; his close concern with it made him the Mahomet of the new dispensation of the astronomical universe, and he was prepared to view all its phenomena in the light of that dispensation. A comet had appeared in 1682—it was still the age when scientific men could think that, by a collision between the earth and a comet, 'this most beautiful order of things would be entirely destroyed and reduced to its ancient chaos'; but this fear was taken as a 'by-the-bye,' which happily interfered with neither observations nor calculations. Observations had duly been made. The data were used to obtain the elements of the orbit, employing Newton's theory as a working hypothesis; and he expresses an incidental regret as to the intrinsic errors of assumed numerical elements and of recorded observations. It then occurred to Halley to calculate similarly the elements of the comet which Kepler and others had seen in 1607, and of which records had been made; the Newtonian theory gave elements in close accord with those belonging to the comet calculated from the latest observations, though a new regret is expressed that the 1607 observations had not been made with more accuracy. On these results he committed himself (being then a man of forty-nine years of age) to a prophecy (which could not be checked for fifty-three years to come) that the comet would return about the end of the year 1758 or the beginning of the next succeeding year; he was willing to leave his conclusion 'to be discussed by the care of posterity, after the truth is found out by the event.' But not completely content with this stage of his work, he obtained with difficulty a book by Apian, giving an account of a comet seen in 1531 and recording a number of observations. Halley, constant to his faith in the Newtonian hypothesis, used that hypothesis to calculate the elements of the orbit of the Apian comet; once more regretting the uncertainty of the data and discounting a very grievous error committed by Apian himself, Halley concluded that the Apian comet of 1531, and the Kepler comet of 1607, and the observed comet of 1682 were one and the same. He confirmed his prediction as to the date of its return, and he

concludes his argument with a blend of confidence and patriotism:—

> Wherefore if according to what we have already said it should return again about the year 1758, candid posterity will not refuse to acknowledge that this was first discovered by an *Englishman*.

Such was Halley's prediction published in the year 1705. The comet pursued its course, and it was next seen on Christmas Day, 1758. Candid posterity, so far from refusing to acknowledge that the discovery was made by an Englishman, has linked Halley's name with the comet, possibly for all time.

We all now could make announcements on the subject of Halley's comet; their fulfilment could be awaited serenely. No vision or inspiration is needed; calculations and corrections will suffice. The comet was seen in 1835, and it is expected again in 1910. No doubt our astronomers will be ready for it; and the added knowledge of electrical science, in connection particularly with the properties of matter, may enable them to review Bessel's often-discussed conjecture as to an explanation of the emission of a sunward tail. But Halley's announcement was made during what may be called the immaturity of the gravitation theory; the realization of the prediction did much to strengthen the belief in the theory and to spread its general acceptance; the crown of conviction was attained with the work of Adams and Leverrier in the discovery, propounded by theory and verified by observation, of the planet Neptune. I do not known an apter illustration of Bacon's dictum that has already been quoted, 'All true and fruitful natural philosophy hath a double scale, ascending from experiments to the invention of causes, and descending from causes to the invention of new experiments.' The double process, when it can be carried out, is one of the most effective agents for the increase of trustworthy knowledge. But until the event justified Halley's prediction, the Cartesian vortex-theory of the universe was not completely replaced by the Newtonian theory; the Cartesian votaries were not at once prepared to obey Halley's jubilant, if stern, injunction to 'leave off trifling * * * with their vortices and their absolute plenum * * * and give themselves up to the study of truth.'

The century that followed the publication of Halley's prediction shows a world that is steadily engaged in the development of the inductive sciences and their applications. Observational astronomy continued its activity quite steadily, reïnforced towards the end of the century by the first of the Herschels. The science of mathematical (or theoretical) astronomy was created in a form that is used to this day; but before this creation could be effected there had to be a development of mathematics suitable for the purpose. The beginnings were made by the Bernoullis (a family that must be of supreme interest to Dr. Francis Galton in his latest statistical compilations, for it contained no fewer than seven mathematicians of mark, distributed over three generations), but the main achievements are due to Euler, Lagrange and Laplace. In particular, the infinitesimal calculus in its various branches (including, that is to say, what we call the differential calculus, the integral calculus, and differential equations) received the development that now is familiar to all who have occasion to work in the subject. When this calculus was developed, it was applied to a variety of subjects; the applications, indeed, not merely influenced, but immediately directed, the development of the mathematics. To this period is due the construction of analytical mechanics at the hands of Euler, d'Alembert, Lagrange and Poisson; but the most significant achieve-

ment in this range of thought is the mathematical development of the Newtonian theory of gravitation applied to the whole universe. It was made, in the main, by Lagrange, as regards the wider theory, and by Laplace, as regards the amplitude of detailed application. But it was a century that also saw the obliteration of the ancient doctrines of caloric and phlogiston, through the discoveries of Rumford and Davy of the nature and relations of heat. The modern science of vibrations had its beginnings in the experiments of Chladni, and, as has already been stated, the undulatory theory of light was rehabilitated by the researches of Thomas Young. Strange views as to the physical constitution of the universe then were sent to the limbo of forgotten ignorance by the early discoveries of modern chemistry; and engineering assumed a systematic and scientific activity, the limits of which seem bounded only by the cumulative ingenuity of successive generations. But in thus attempting to summarize the progress of science in that period, I appear to be trespassing upon the domains of other sections; my steps had better be retraced so as to let us return to our own upper air. If I mention one more fact (and it will be a small one), it is because of its special connection with the work of this section. As you are aware, the elements of Euclid have long been the standard treatise of elementary geometry in Great Britain; and the Greek methods, in Robert Simson's edition, have been imposed upon candidates in examination after examination. But Euclid is on the verge of being disestablished; my own University of Cambridge, which has had its full share in maintaining the restriction to Euclid's methods, and which was not uninfluenced by the report of a committee of this association upon the subject, will, some six or seven weeks hence, hold its last examination in which those methods are prescriptively required. The disestablishment of Euclid from tyranny over the youthful student on the continent of Europe was effected before the end of the eighteenth century.

But it is time for me to pass on to the third of the centenaries, with which the present year can be associated. Not so fundamental for the initiation of modern science as was the year in which the 'Advancement of Learning' was published, not so romantic in the progress of modern science as was the year in which Halley gave his prediction to the world, the year 1805 (turbulent as it was with the strife of European politics) is marked by the silent voices of a couple of scientific records. In that year Laplace published the last progressive instalment of his great treatise on 'Celestial Mechanics,' the portion that still remained for the future being solely of an historical character; the great number of astronomical phenomena which he had been able to explain by his mathematical presentation of the consequences of the Newtonian theory would, by themselves, have been sufficient to give confidence in the validity of that theory. In that year also Monge published his treatise, classical and still to be read by all students of the subject, 'The Application of Algebra to Geometry'; it is the starting point of modern synthetic geometry, which has marched in ample development since his day. These are but landmarks in the history of mathematical science, one of them indicating the completed attainment of a tremendous task, the other of them initiating a new departure; both of them have their significance in the progress of their respective sciences.

When we contemplate the activity and the achievements of the century that has elapsed since the stages which have just been mentioned were attained in mathematical science, the amount, the variety, the

progressive diligence, are little less than bewildering. It is not merely the vast development of all the sciences that calls for remark: no less striking is their detailed development. Each branch of science now has an enormous array of workers, a development rendered more easily possible by the growing increase in the number of professional posts; and through the influence of these workers and their labors there is an ever-increasing body of scientific facts. Yet an aggregate of facts is not an explanatory theory any more necessarily than a pile of carefully fashioned stones is a cathedral; and the genius of a Kepler and a Newton is just as absolutely needed to evolve the comprehending theory as the genius of great architects was needed for the Gothic cathedrals of France and of England. Not infrequently it is difficult to make out what is the main line of progress in any one subject, let alone in a group of subjects; and though illumination comes from striking results that appeal, not merely to the professional workers, but also to unprofessional observers, this illumination is the exception rather than the rule. We can allow, and we should continue to allow, freedom of initiative in all directions. That freedom sometimes means isolation, and its undue exercise can lead to narrowness of view. In spite of the complex ramification of the sciences which it has fostered, it is a safer and a wiser spirit than that of uncongenial compulsion, which can be as dogmatic in matters scientific as it can be in matters theological. Owing to the varieties of mind, whether in individuals or in races, the progress of thought and the growth of knowledge are not ultimately governed by the wishes of any individual or the prejudices of any section of individuals. Here, a school of growing thought may be ignored; there, it may be denounced as of no importance; somewhere else, it may be politely persecuted out of possible existence. But the here, and the there, and the somewhere else do not make up the universe of human activity; and that school, like Galileo's earth in defiance of all dogmatic authority, still will move.

This complete freedom in the development of scientific thought, when the thought is applied to natural phenomena, is all the more necessary because of the ways of nature. Physical nature cares nothing for theories, nothing for calculations, nothing for difficulties, whatever their source; she will only give facts in answer to our questions, without reasons and without explanations; we may explain as we please and evolve laws as we like, without her help or her hindrance. If from our explanations and our laws we proceed to prediction, and if the event justifies the prediction through agreement with recorded fact, well and good: so far we have a working hypothesis. The significance of working hypotheses, in respect of their validity and their relation to causes, is a well-known battle-ground of dispute between different schools of philosophers; it need not detain us here and now. On the other hand, when we proceed from our explanations and our laws to a prediction, and the prediction in the end does not agree with the fact to be recorded, it is the prediction that has to give way. But the old facts remain and the new fact is added to them; and so facts grow until some working law can be extracted from them. This accumulation of facts is only one process in the solution of the universe: when the compelling genius is not at hand to transform knowledge into wisdom, useful work can still be done upon them by the construction of organized accounts which shall give a systematic exposition of the results, and shall place them as far as may be in relative significance.

Let me pass from these generalities, which have been suggested to my mind by the consideration of some of the scientific changes that have taken place during the last hundred years, and let me refer briefly to some of the changes and advances which appear to me to be most characteristic of that period. It is not that I am concerned with a selection of the most important researches of the period. Estimates of relative importance are often little more than half-concealed expressions of individual preferences or personal enthusiasms; and though each enthusiastic worker, if quite frank in expressing his opinion, would declare his own subject to be of supreme importance, he would agree to a compromise that the divergence between the different subjects is now so wide as to have destroyed any common measure of comparison. My concern is rather with changes, and with tendencies where these can be discerned.

The growth of astronomy has already occupied so large a share of my remarks that few more words can be spared here. Not less, but more, remarkable than the preceding centuries in the actual exploration of the heavens, which has been facilitated so much by the improvements in instruments and is reinforced to such effect by the cooperation of an ever-growing band of American astronomers, it has seen a new astronomy occupy regions undreamt of in the older days. New methods have supplemented the old; spectroscopy has developed a science of physics within astronomy; and the unastronomical brain reels at the contents of the photographic chart of the heavens which is now being constructed by international cooperation and will, when completed, attempt to map ten million stars (more or less) for the human eye.

Nor has the progress of physics, alike on the mathematical side and the experimental side, been less remarkable or more restricted than that of astronomy. The elaborate and occasionally fantastic theories of the eighteenth century, in such subjects as light, heat, even as to matter itself, were rejected in favor of simpler and more comprehensive theories. There was one stage when it seemed as if the mathematical physicists were gradually overtaking the experimental physicists; but the discoveries in electricity begun by Faraday left the mathematicians far behind. Much has been done towards the old duty, ever insistent, of explaining new phenomena; and the names of Maxwell, Weber, Neumann, and Hertz need only to be mentioned in order to suggest the progress that has been made in one subject alone. We need not hesitate to let our thoughts couple, with the great physicists of the century, the leaders of that brilliant band of workers upon the properties of matter who carry us on from wonder to wonder with the passage of each successive year.

Further, it has been an age when technical applications have marched at a marvelous pace. So great has been their growth that we are apt to forget their comparative youth; yet it was only the middle of the century which saw the awakening from what now might be regarded as the dark ages. Nor is the field of possible application nearing exhaustion: on the contrary, it seems to be increasing by reason of new discoveries in pure science that yet will find some beneficent outcome in practice. Invisible rays and wireless telegraphy may be cited as instances that are occupying present activities, not to speak of radium, the unfolding of whose future is watched by eager minds.

One gap, indeed, in this subject strikes me. There are great histories of mathematics and great histories of astronomy; I can find no history of physics on the grand scale. Some serviceable manuals there are,

as well as monographs on particular topics; what seems to me to be lacking is some comprehensive and comparative survey of the whole range. The history of any of the natural sciences, like the history of human activity, is not merely an encyclopædic record of past facts; it reveals both the spirit and the wealth which the past has bequeathed to the present, and which, in due course, the present will influence before transmission to the future. Perhaps all our physicists are too busy to spare the labor needed for the production of a comprehensive history; yet I cannot help thinking that such a contribution to the subject would be of great value, not to physicists alone.

But, as you hear me thus referring to astronomy and to physics, some of you may think of the old Roman proverb which made the cobbler not to look above his last; so I take the opportunity of referring very briefly to my own subject. One of the features of the century has been the continued development of mathematics. As a means of calculation the subject was developed as widely during the earlier portion of the century as during the preceding century; it soon began to show signs of emergence as an independent science, and the later part of the century has witnessed the emancipation of pure mathematics. It was pointed out, in connection with the growth of theoretical astronomy, that mathematics developed in the direction of its application to that subject. When the wonderful school of French physicists, composed of Monge, Carnot, Fourier, Poisson, Poinsot, Ampère and Fresnel (to mention only some names), together with Gauss, Kirchhoff and von Helmholtz in Germany, and Ivory, Green, Stokes, Maxwell and others in England, applied their mathematics to various branches of physics, for the most part its development was that of an ancillary subject. The result is the superb body of knowledge that may be summarized under the title of 'mathematical physics'; but the final interest is the interest of physics, though the construction has been the service of mathematics. Moreover, this tendency was deliberate, and was avowed in no uncertain tone. Thus Fourier could praise the utility of mathematics by declaring that 'there was no language more universal or simpler, more free from errors or obscurity, more worthy of expressing the unchanging relations of natural entities'; in a burst of enthusiasm he declares that, from the point of view he had indicated, 'mathematical analysis is as wide as nature herself,' and 'it increases and grows incessantly stronger amid all the changes and errors of the human mind.' Mathematicians might almost blush with conscious pleasure at such a laudation of their subject from such a quarter, though it errs by both excess and defect; but the exultation of spirit need not last long. The same authority, when officially expounding to the French Academy the work of Jacobi and of Abel upon elliptic functions, expressed his chilling opinion (it had nothing to do with the case) that 'the questions of natural philosophy, which have the mathematical study of all important phenomena for their aim, are also a worthy and principal subject for the meditations of geometers. It is to be desired that those persons who are best fitted to improve the science of calculation should direct their labors to these important applications.' Abel was soon to pass beyond the range of admonition; but Jacobi, in a private letter to Legendre, protested that the scope of the science was not to be limited to the explanation of natural phenomena. I have not quoted these extracts by way of even hint of reproach against the author of such a wonderful creation as Fourier's analytical theory

of heat; his estimate could have been justified on a merely historical review of the circumstances of his own time and of past times; and I am not sure that his estimate has not its exponents at the present day. But all history shows that new discoveries and new methods can spread to issues wider than those of their origins, and that it is almost a duty of human intelligence to recognize this possibility in the domain of progressive studies. The fact is that mathematical physics and pure mathematics have given much to each other in the past and will give much to each other in the future; in doing so, they will take harmonized action in furthering the progress of knowledge. But neither science must pretend to absorb the activity of the other. It is almost an irony of circumstance that a theorem, initiated by Fourier in the treatise just mentioned, has given rise to a vast amount of discussion and attention, which, while of supreme value in the development of one branch of pure mathematics, have hitherto offered little, if anything, by way of added explanation of natural phenomena.

The century that has gone has witnessed a wonderful development of pure mathematics. The bead-roll of names in that science—Gauss; Abel, Jacobi; Cauchy, Riemann, Weierstrass, Hermite; Cayley, Sylvester; Lobatchewsky, Lie—will on only the merest recollection of the work with which their names are associated show that an age has been reached where the development of human thought is deemed as worthy a scientific occupation of the human mind as the most profound study of the phenomena of the material universe.

The last feature of the century that will be mentioned has been the increase in the number of subjects, apparently dissimilar from one another, which are now being made to use mathematics to some extent. Perhaps the most surprising is the application of mathematics to the domain of pure thought; this was effected by George Boole in his treatise 'Laws of Thought,' published in 1854; and though the developments have passed considerably beyond Boole's researches, his work is one of those classics that mark a new departure. Political economy, on the initiative of Cournot and Jevons, has begun to employ symbols and to develop the graphical methods; but there the present use seems to be one of suggestive record and expression, rather than of positive construction. Chemistry, in a modern spirit, is stretching out into mathematical theories; Willard Gibbs, in his memoir on the equilibrium of chemical systems, has led the way; and, though his way is a path which chemists find strewn with the thorns of analysis, his work has rendered, incidentally, a real service in coordinating experimental results belonging to physics and to chemistry. A new and generalized theory of statistics is being constructed; and a school has grown up which is applying them to biological phenomena. Its activity, however, has not yet met with the sympathetic good-will of all the pure biologists; and those who remember the quality of the discussion that took place last year at Cambridge between the biometricians and some of the biologists will agree that, if the new school should languish, it will not be for want of the tonic of criticism.

If I have dealt with the past history of some of the sciences with which our section is concerned, and have chosen particular epochs in that history with the aim of concentrating your attention upon them, you will hardly expect me to plunge into the future. Being neither a prophet nor the son of a prophet, not being possessed of the knowledge which enabled Halley to don the prophet's mantle with confidence, I shall

venture upon no prophecy even so cautious as Bacon's—'As for the mixed mathematics, I may only make this prediction, that there can not fail to be more kinds of them as nature grows further disclosed'—a declaration that is sage enough, though a trifle lacking in precision. Prophecy, unless based upon confident knowledge, has passed out of vogue, except perhaps in controversial politics; even in that domain, it is helpless to secure its own fulfilment. Let me rather exercise the privilege of one who is not entirely unfamiliar with the practice of geometry, and let me draw the proverbial line before indulgence in prophetic estimates. The names that have flitted through my remarks, the discoveries and the places associated with those names, definitely indicate that, notwithstanding all appearance of divergence and in spite of scattered isolation, the sum of human knowledge, which is an inheritance common to us all, grows silently, sometimes slowly, yet (as we hope) safely and surely, through the ages. You who are in South Africa have made an honorable and an honored contribution to that growing knowledge, conspicuously in your astronomy and through a brilliant succession of astronomers. Here, not as an individual, but as a representative officer of our brotherhood in the British Association, I can offer you no better wish than that you may produce some men of genius and a multitude of able workers who, by their researches in our sciences, may add to the fame of your country and will contribute to the intellectual progress of the world.

A. R. FORSYTH.

SCIENTIFIC BOOKS.

Catalogue of the Lepidoptera Phalænæ in the British Museum, London. Vol. IV., Noctuidæ (part), 1903; Vol. V., Noctuidæ (continued), 1905. By Sir GEORGE F. HAMPSON, Bart.

This is a continuation of the monographs of the moths of the world, of which Vol. III. was noticed in SCIENCE, N. S., XV., 99, 1901. A notice of Vol. IV. will be found in the *Canadian Entomologist,* XXXVI., 27, 1904. Volume V., now before us, consists of 634 pages and treats of 2,073 species of Noctuidæ, comprising the subfamily Hadeninæ. These moths have unspined hind tibiæ and hairy eyes, and are familiar to us under the name *Mamestra* and allies. But these familiar names are again largely changed, unavoidably, no doubt, but we fear that the changes are not permanent. Even if subsequent authors can be induced to respect Sir George Hampson's selections of the types of the older genera, we doubt if he will be generally followed in defining no genera on secondary sexual characters. This is done generally in other families of Lepidoptera and the characters prove very useful. We think some of the genera as used in the volume before us would stand subdivision, *Polia,* for example, which contains 209 species. This would save the old genus *Mamestra,* which now sinks as a synonym of *Polia.* These remarks apply to the other volumes as well and are a criticism on the general system adopted. It is not to be expected that the system could be changed during the progress of the work.

A number of our North American species, particularly those recently described, sink as synonyms. This is mostly perfectly justified, as there has been a tendency recently to describe too many forms as species in the Noctuidæ. This tendency has received a just rebuke.

On page 24, *Scotogramma* is marked as a 'new' genus, no doubt by an oversight.

On page 178 all the forms of *comis* and *olivacea* fall together into the synonymy. I believe this is going a little too far, as I think there can be distinguished two species, though closely allied. Otherwise my contention about these forms is sustained.

On page 267 the name *Chabuata velutina* is used. It should be *Chabuata lutina. Velutina* was preoccupied when described and the author very properly changed the name. The